# Sequence and Series

## Sequence

Given a sequence (dãy số) , the sequence is said to be convergence if and only if

|  |  |  |
| --- | --- | --- |
|  |  |  |

where is a finite number, vice versa.

An increasing sequence which is upper bounded or a decreasing sequence which is lower bounded is said to be convergent sequence.

## Series

### Definition

Series (chuỗi số) is a summation of a particular sequence , symbolically it can be expressed by

|  |  |  |
| --- | --- | --- |
|  |  |  |

There are two fundamental convergent series which are -series and geometric series (power series) with a given constraint of convergence as follows:

* -series

|  |  |  |
| --- | --- | --- |
|  |  |  |

* Geometric series

|  |  |  |
| --- | --- | --- |
|  |  |  |

### 7-tests for Series

**i. Divergence test**

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| --- | --- | --- |
|  |  |  |

If a limit of a sequence is not zero, or does not exists, then its sequence is divergence.

*(Test này cho chúng ta biết rằng nếu tính lim ra khác 0 thì ngay lập tức series của mình divergent. Test này được dùng ĐẦU TIÊN khi thao tác với chuổi)*

**ii. Integral test**

Given a function be a positive monotonic decreasing function on the interval and then

|  |  |  |
| --- | --- | --- |
|  |  |  |

It holds that

|  |  |  |
| --- | --- | --- |
| 1. |  |  |
| 2. |  |  |

*(Nếu có hàm f dương, đơn điệu giảm (positive monotonic decreasing function) thể hiện cho dãy , khi đó integral test cho ta Eq 1.4. Khi đó nếu tích phân hội tụ thì series tương ứnghội tụ và ngược lại. Test này ÍT DÙNG)*

**iii. Comparison test**

Suppose that we have two sequence such that then,

|  |  |  |
| --- | --- | --- |
| 1. |  |  |
| 2. |  |  |

*(Test này nói rằng: lớn hội tụ nhỏ hội tụ / nhỏ phân kì lớn phân kỳ. Test này cực kì HỮU DỤNG)*

**iv. Limit comparison test**

Suppose that we have two sequence such that if the limit of ratio of two sequence such that

|  |  |  |
| --- | --- | --- |
|  |  |  |

then, it leads to

|  |  |  |
| --- | --- | --- |
| 1. |  |  |
| 2. |  |  |

*(Khi L là số dương hữu hạng thì và cùng tình chất. Test này ÍT DÙNG)*

**v. Alternating series test**

Suppose that we have a sequence or , where is positive sequence for all . If is decreasing sequence and

|  |  |  |
| --- | --- | --- |
|  |  |  |

then the series of is convergence.

*(Nếu ta có một dãy đan dấu mà trị tuyệt đối của nó giảm và hội tụ về 0 thi chuỗi cũa dãy đó hội tụ. Test này ÍT DÙNG)*

**vi & vii. Ratio test and Root test**

Ratio test

|  |  |  |
| --- | --- | --- |
|  |  |  |

Root test

|  |  |  |
| --- | --- | --- |
|  |  |  |

Consider the value of in the 3 following cases:

* : Absolute convergence.
* : No conclusion.
* : Divergence.

*(Hai test này cực kì HỮU DỤNG và có dấu hiệu đặc trưng riêng. Có giai thừa thì ratio test, có mũ thì root test)*

### Absolute Convergence and Conditional Convergence

Given a sequence , it always satisfies the following condition

|  |  |  |
| --- | --- | --- |
|  |  |  |

Consequence of comparison test give us if sequence of is convergence then, sequence of also converges.

*(Nếu chuổi của hội tụ thì chuỗi của cũng hội tụ)*

|  |  |
| --- | --- |
| Conditional convergence | Absolute convergence |
|  |  |

**Consequence:** If a sequence is convergence due to Alternating series test, Ratio test and Root test the sequence must be absolute convergence.

*(Nếu sử dụng Alternating series test, Ratio test và Root test cho ra chuổi hội tụ thì chuổi đó phải là hội tụ tuyệt đối)*

## Power series

Given a power series in the following form

|  |  |  |
| --- | --- | --- |
|  |  |  |

Where is center of expansion (expand of function about point c)

If we found that such that the series is convergence then, is called **radius of convergent**. It leads to is the open interval of convergence, we have to additionally check for convergence at the at two endpoints to fully find the interval of convergence.

*(Nếu tìm được sao cho chuỗi đã cho hội tụ thì R được gọi là radius of convergent. Sau đó check thêm 2 đầu mút của khoảng để tìm interval of convergence)*

*(Root test và ratio test thường được sử dụng trong bài toán này)*

# Geometry of Space

**Note:** All bold notations are vector, for example,

## Vector Space

### Vector Calculus

|  |  |
| --- | --- |
| **Name** | **Operator** |
| Dot product |  |
| Cross product |  |
|  |  |
| Projection of along vector |  |
|  |  |
| Del – Gradient vector |  |

### Line Equation

Passing through point with direction vector

*(Đi qua điểm A và có vector chỉ phương )*

|  |  |  |  |
| --- | --- | --- | --- |
| Parametric equation | | Symmetric equation | |
|  |  |  |  |

### Plane Equation

Passing through point with normal vector

*(Đi qua điểm A và có vector pháp tuyến )*

|  |  |  |
| --- | --- | --- |
|  |  |  |

or

|  |  |  |
| --- | --- | --- |
|  |  |  |

Distant from a point to plane

|  |  |  |
| --- | --- | --- |
|  |  |  |

(Khoảng cách từ điểm tới mặt phẳng )

## Coordinates Conversion

### Cylindrical Coordinate Systems

|  |  |
| --- | --- |
| Coordinate conversion | Vector conversion |
|  |  |

Differential vector:

### Spherical Coordinate Systems

|  |  |
| --- | --- |
| Coordinate conversion | Vector conversion |
|  |  |

Differential vector:

## Vector function

Given a vector function in rectangular form

|  |  |  |
| --- | --- | --- |
|  |  |  |

Its differential vector is given by

|  |  |  |
| --- | --- | --- |
|  |  |  |

Similarly for integration.

Arc length of a function

|  |  |  |
| --- | --- | --- |
|  |  |  |

Arc length of a vector function

|  |  |  |
| --- | --- | --- |
|  |  |  |

# Partial Derivative

## Introduction

|  |  |  |
| --- | --- | --- |
| Scalar function: |  |  |
| Vector field: |  |  |

Del operator

|  |  |  |
| --- | --- | --- |
|  |  |  |

**Consequence:**

## Chain Rule

Given that: where , the chain rule gives us

|  |  |  |
| --- | --- | --- |
|  |  |  |

Given that: where

|  |  |  |
| --- | --- | --- |
|  |  |  |

## Implicit Derivative

Given that:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Given that:

|  |  |  |
| --- | --- | --- |
|  |  |  |

## Curl and Divergence

Curl

|  |  |  |
| --- | --- | --- |
|  |  |  |

Divergence

|  |  |  |
| --- | --- | --- |
|  |  |  |

Consequence

|  |  |  |
| --- | --- | --- |
|  |  |  |

## Directional Derivative and Gradient Vector

Gradient vector

|  |  |  |
| --- | --- | --- |
|  |  |  |

Given: at and **unit** directional vector . The directional derivative is given by

|  |  |  |
| --- | --- | --- |
|  |  |  |

Maximum directional derivative

|  |  |  |
| --- | --- | --- |
|  |  |  |

## Tangent Plane and Normal Line

Given a surface: and a point

Tangent plane at point of surface

|  |  |  |
| --- | --- | --- |
|  |  |  |

Normal line passing through point of surface

|  |  |  |
| --- | --- | --- |
|  |  |  |

# Maximum and Minimum Problem

## Local Min-Max Problem

**Note:** Critical point(s) is the point at which its first derivative equals to zero or undefined.

Consider at critical points

* is local minimum.
* is local maximum.
* is a saddle point.
* give nothing.

## Lagrange Multipliers

Given that: with constrains

Solve the system of equations below to find the absolute min-max

|  |  |  |
| --- | --- | --- |
|  |  |  |

## Solving a Particular Min-Max Problem

Depending on the requirements of problem, we separate a particular problem into 4 cases as follows:

* Case 1: Find local min-max points: apply (1)
* Case 2: Find absolute min-max with constrains: apply (2)
* Case 3: Find absolute min-max on the region D (simple shape): apply (1) after that substitute the boundary into and continue to find abs min-max
* Case 4: Find absolute min-max on the region D (complex shape): apply (1) and (2).

# Multiple Integral

## Line Integral

Given that: or

Scalar integral

Vector field integral

Arc length

## Surface Integral

Given that:

Simple case:

Scalar integral (general case):

Scalar integral (simple case):

Vector field integral (general case):

Vector field integral (simple case):

Surface area for surface

Surface area for surface

## Frequently Used Theorem

Stoke’s theorem

|  |  |  |
| --- | --- | --- |
|  |  |  |

(C must be a closed path)

Divergence theorem

|  |  |  |
| --- | --- | --- |
|  |  |  |

(S must be a closed surface)

Green’s theorem

|  |  |  |
| --- | --- | --- |
|  |  |  |

(C must be a closed path and positive oriented)

**Note:**

* .
* .
* Unit vector